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Reply by Authors to D. L. Clingman and T. L. Rosebrock

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In a previous paper,[1] the authors wanted to illustrate that the simple ballistic pendulum is subject to error, and effects such as were observed should be considered before accepting impulse data obtained with pendulums. It was not intended to condemn ballistic pendulums per se; in fact, it was hoped to apply this technique in the authors' present work.

The solution to the ablation problem proposed by D. L. Clingman and T. L. Rosebrock seems to be a good one. Another approach simply is to reduce the energy density of the plasma at the surface of the pendulum by moving the pendulum away from the plasma accelerator so that the beam can spread radially and axially; however, the pendulum must become larger to collect all of the beam.

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Comment on "Invariant Two-Body Velocity Components"

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In a previous paper,[1] Cronin and Schwartz discussed the invariant two-body velocity components of orbital body motion. Speculation was presented on possible useful applications of this property of invariance to the solution of two-body trajectory problems.

Since about three years ago, various studies of such applications have been conducted and are continuing. As noted in a XIIIth International Astronautical Congress paper,[2] additional literature on such applications is available.[3–14] Also, Newton's paper[15] is based upon this invariance property.

In general, the invariance of the specified orbital velocity components describes a circle that defines the orbital velocity vector in inertial space. This figure is known as the velocity hodograph, originally discovered by Hamilton and Möbius.

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Explicit Solution of the "Three-Moments Equation"

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The solution of the "three-moments equation" given by the author[1] requires the search for eigenvalues and eigenvectors of a certain operator. To remove these difficulties and to find the explicit solution, one can substitute, instead of a finite-difference operator and boundary conditions, a new operator that transforms the boundary-value problem into an algebraic equation, without previously solving the problem. Commutation properties of this operator with Green's function make it possible to find the solution in classical form.

The "three-moments problem" has the notation

$$L_{mn}M_n = -G_m$$
$$M_0 = M_N = 0 \qquad n = 0,1,\ldots,N \tag{1}$$

where the finite-difference operator **L** and boundary conditions can be written as follows:

$$\left.\begin{aligned} L_{mn} &= \delta_{m+1,n} + 4\delta_{m,n} + \delta_{m-1,n} \\ M_0 &= M_N = 0 \end{aligned}\right\} \quad n = 0,1,\ldots,N \tag{2}$$

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